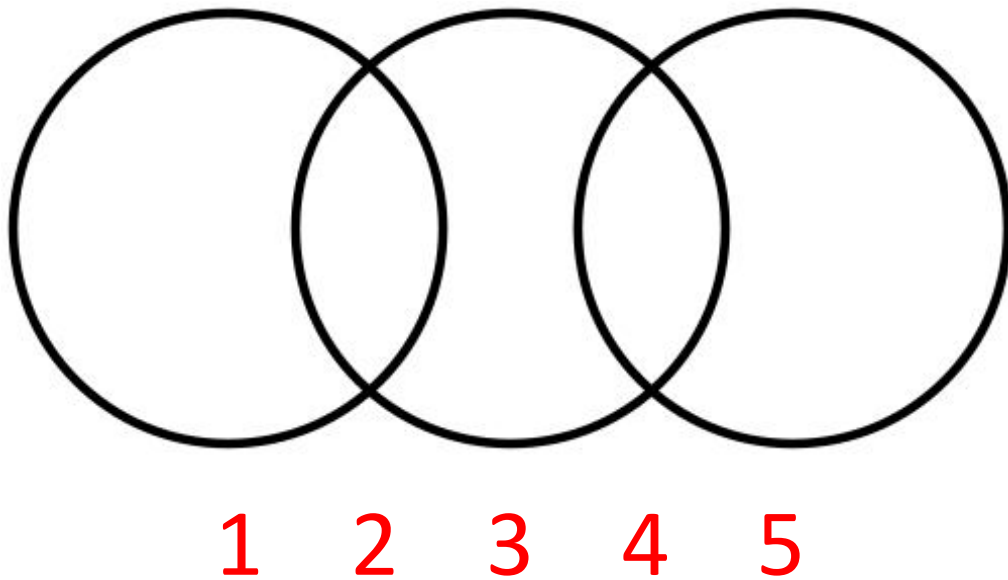


# Puzzle of the Week

## *Equal Sums – 1*

**THE CHALLENGE:** Here is a diagram created by overlapping three circles. The overlapping circles create five regions. Put a number in each of the five regions, using each of the numbers 1 to 5 exactly once, so that the sum of the numbers in each circle is the same.



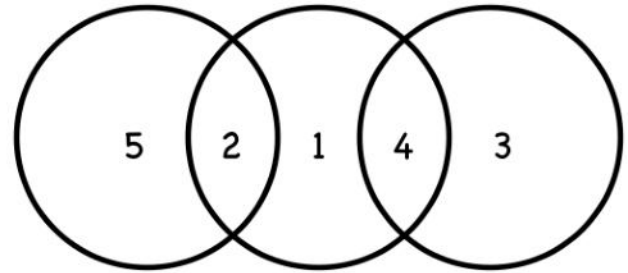
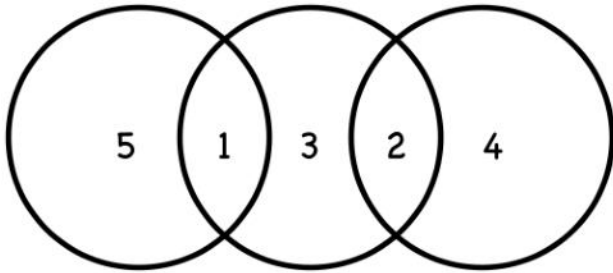
**EXPLORATION:** How many different answers can you find? How do you know if you have found them all?

# Puzzle of the Week

## *Equal Sums – 1 – Notes*

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**THE CHALLENGE & EXPLORATION:** There are two solutions. Going from left to right, the sum in each circle is 6 and 7.



Analyze the possibilities by letting  $A$  and  $B$  be the two numbers in the intersections of the circles. Let  $\text{Sum}$  be the common sum inside each circle. Then  $3 \times \text{Sum} = 1 + 2 + 3 + 4 + 5 + A + B = 15 + A + B$ .

The left side of  $3 \times \text{Sum} = 15 + A + B$  is a multiple of 3, so the right side is as well. This forces  $A + B$  to be a multiple of three. That leaves only three possibilities.

- $A + B = 3$ . In this case  $3 \times \text{Sum} = 15 + 3 = 18$  tells us  $\text{Sum} = 6$ , and  $A$  and  $B$  are 1 and 2.
- $A + B = 6$ . In this case  $3 \times \text{Sum} = 15 + 6 = 21$  tells us  $\text{Sum} = 7$ .  $A + B = 6$  forces  $A$  and  $B$  to be either 1 and 5 or 2 and 4. Having  $A$  and  $B$  be 1 and 5 does not work (1 is repeated), so that leaves us with just 2 and 4.
- $A + B = 9$ . In this case  $3 \times \text{Sum} = 15 + 9 = 24$  tells us  $\text{Sum} = 8$ , and  $A$  and  $B$  are 4 and 5. However, because  $A$  and  $B$  are both in the middle circle, it is not possible for  $A + B = 9$  and yet the  $\text{Sum}$  is only 8. So this case cannot happen.